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Classical Estimation of Mixed Rayleigh Distribution in Type I Progressive Censored

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Abstract

This article is considered with the problem of estimating the parameters under type I Progressive censored, a new statistical model called mixed Rayleigh distribution is suggested; the maximum likelihood estimates for its unknown parameters and their approximate asymptotic variance covariance matrix are derived. An iterative procedure is developed and tested numerical example to obtain the new estimators and their variance covariance matrix. A Monte Carlo simulation is used to investigate the accuracy of this estimator and an example is given to illustrate the maximum likelihood estimate.

Key Words and Phrases: *Finite mixture; type I progressive censored; Rayleigh distribution; maximum likelihood estimation; asymptotic variance covariance matrix*.

1. Introduction

In the most simple life test we have one population and only one type of failure. For every item put on test we have the time it took for the failure to occur; we will denote the corresponding failure time density as $f_i(x)$. Other; more general models are proposed and contrasted by Cox (1959). Thus we can also consider a life test in which the items in the single population are subject to more than one type of failure, which will be referred to as a competing risk situation. This model is realized in medical and actuarial work where the estimation and comparison of death rates from a particular cause require correction for deaths from other causes; or in tensile strength testing where there may be two or more types of failure, e.g., jaw breaks and fractures in the test specimen [see Oppenheimer (1971)]. Another generalization arises if we consider a situation in which we have s distinct populations, and the probability that an item is in the i^{th} population is p_i, $0 \le p_i \le 1$, $\sum_{i=1}^{s} p_i = 1$. An item which is in population i is subject only to the risk of failure of the i^{th} type, which can be represented by a failure time density of the form $f_i(x)$. The density corresponding to this life test will be referred to as a mixture of densities and expressed as

$$f(x) = \sum_{i=1}^{s} p_i f_i(x)$$

One of the important families of distributions in lifetime tests is the Rayleigh distribution with probability density functions

$$f_i(x;\beta_i) = \beta_i.x.e^{-\frac{\beta_i}{2}x^2} \qquad x \ge 0, \beta_i \ge 0, i = 1,2,...s \tag{1}$$

and the survival functions is given by

$$R_i(x;\beta_i) = e^{-\frac{\beta_i}{2}x^2} \qquad x \ge 0, \beta_i \ge 0, i = 1,2,...s \tag{2}$$

When the $y = x^2$ in (1.1) and (2) give the probability density function and the cumulative distribution of Exponential distribution with scale parameter $\beta/2$.

Mixed model is encountered in many fields of applied science, when population is not homogeneous, but is made-up from sub-populations, mixed in unknown proportions. In the present situation, individuals are of two types ($s = 2$), the probability that an individual is of the first type being p_1 and probability that it is of second type is p_2, where $p_1 + p_2 = 1$. An individual of type I is subject only to the risk of failure of the first type. The probability density function and survival function of the mixed distribution will be

$$f(x) = p_1 f_1(x) + p_2 f_2(x) \tag{3}$$

$$R(x) = p_1 R_1(x) + p_2 R_2(x) \tag{4}$$

By using equations (1) and (2) in (3) and (4), probability density function and the survival function of mixed Rayleigh distribution will be

$$f(x;\beta_1,\beta_2) = p_1\beta_1.x.e^{-\frac{\beta_1}{2}x^2} + p_2\beta_2.x.e^{-\frac{\beta_2}{2}x^2} \qquad x \ge 0, \beta_i \ge 0, \forall i. \tag{5}$$

and

$$R(x;\beta_1,\beta_2) = p_1 e^{-\frac{\beta_1}{2}x^2} + p_2 e^{-\frac{\beta_2}{2}x^2} \qquad x \ge 0, \beta_i \ge 0, \forall i. \tag{6}$$

respectively.

We now have the censoring occurring progressively which defined as: n units are put on life test at time zero. At times $T_1, T_2, ..., T_m$ numbers $k_1, k_2, ..., k_{m-1}$ of failure units are occurs, also predetermined numbers $r_1, r_2, ..., r_m$ of live units are removed from experimentation, respectively. If the time of removal are fixed with T_m being the time of experiment termination and r_m being the number of surviving units at that time. Where $k = k_1 + k_2 + ... + k_m$ .The likelihood function will be

$$L(X;\beta_1,\beta_2) = C\left(\prod_{i=1}^{k} f(x_{(i)};\beta_1,\beta_2)\right)\prod_{j=1}^{m}(R(T_j;\beta_1,\beta_2))^{r_j} \tag{7}$$

where C is a constant, k is the number of failures, $x_{(i)}$ is the lifetime of the i^{th} order statistic and $f(x_{(i)};\beta_1,\beta_2), R(T_j;\beta_1,\beta_2)$ are the density function and the survival function in (5) and (6) respectively. Progressive censoring schemes are carried out under the assumption that continuous monitoring is in place, if $r_1 = r_2 = ... = r_{m-1} = 0$, progressive censoring type I reduces to single censoring type I.

Most of literatures confined its attention to just on Type I censored which will draw from mixed model; for example, Mendenhall and Hader (1958) derived the maximum likelihood estimators for the unknown parameters of the mixed exponential model using type I censored and studied their properties. Oppenheimer (1971) estimated the parameter of mixed exponential from complete and censored samples. Jones and Ashour (1976) discussed the same estimation problem using Bayesian approach, on characterization of mixtures were studied by, Nassar and Mahmoud

(1985), Nassar (1988), Gharib (1996) and Ismail and ElKhodary (2001). Many author interested with inferences on mixtures of exponential distributions among them Rider (1961), Everitt and Hand (1981), Al-Hussaini (1999), Bartoszewicz (2002) and Jaheen (2005). Radhakrishna et al. (1992) derived maximum likelihood estimators of the parameters of two component mixture generalized gamma distribution. Also, Ashour and Abd-el Hafez (1984) and Ashour (1985) estimated the parameters of a Weibull exponential using maximum likelihood and Bayesian method for type I censored samples. Also, Elsherpieny (2007) and Shawky and Bakoban (2009) estimated the parameters of mixed generalized exponentionally and Exponentiated Gamma distributions respectively. Mixture of Exponentiated Pareto and Exponential Distribution was studied by Hanaa and Abu-Zinadah (2010).

In this study we shall confine our attention to just in introduce progressively type I censored sample and derive the maximum likelihood estimates for unknown parameters of two populations under this type; both of them will be have Rayleigh distributions. The asymptotic variance covariance matrix was obtained by taking the inverse of the information matrix, which required numerical integration. The organization of the paper is as follows: Section 2 maximum likelihood estimators are discussed. Asymptotic variance covariance matrix under progressively type I censored sample will derived in section 3. In section 4 an example will be discussed to illustrate the application of results. Finally, Conclusions are given in Section 5.

2. Maximum likelihood estimators

Now we have the censoring occurring progressively in m stages at time T_i, such that $T_i > T_{i-1}$. $i = 1,2,...,m$, and at the i^{th} stage of censoring r_i units, selected at random from the survivors, are withdrawn from the test. By using (1.5) and (1.6) in expression (1.7) and to log likelihood can be written as:

$$\log L = \sum_{i=1}^{k} \log\left[p_1\beta_1 x_{(i)} e^{\frac{-\beta_1}{2}x_{(i)}^2} + p_2\beta_2 x_{(i)} e^{\frac{-\beta_2}{2}x_{(i)}^2} \right] + \sum_{i=1}^{m} r_i \log\left[p_1 e^{\frac{-\beta_1}{2}T_i^2} + p_2 e^{\frac{-\beta_2}{2}T_i^2} \right] \quad (8)$$

Taking partial derivatives of (8), we obtain the following maximum likelihood estimating equations;

$$\frac{\partial \log L}{\partial p_1}=\sum_{i=1}^{k}\left[\frac{\left(\beta_1 x_{(i)} e^{\frac{-\beta_1}{2}x_{(i)}^2}-\beta_2 x_{(i)} e^{\frac{-\beta_2}{2}x_{(i)}^2}\right)}{f(x_{(i)};\beta_1,\beta_2)}\right]+\sum_{i=1}^{m} r_i\left[\frac{\left(e^{\frac{-\beta_1}{2}T_i^2}-e^{\frac{-\beta_2}{2}T_i^2}\right)}{R(x;\beta_1,\beta_2)}\right] \qquad (9)$$

$$\frac{\partial \log L}{\partial \beta_j}=\sum_{i=1}^{k}\left\{p_j\,\frac{e^{\frac{-\beta_j}{2}x_{(i)}^2}\left(2x_{(i)}-\beta_j x_{(i)}^3\right)}{2\,f(x_{(i)};\beta_1,\beta_2)}\right\}-\sum_{i=1}^{m}\left[\frac{r_i.p_j.T_i^2 e^{\frac{-\beta_j}{2}T_i^2}}{2.R(x;\beta_1,\beta_2)}\right] \qquad (10)$$

Multiplying equation (9) by p_1 and adding

$$\sum_{i=1}^{k}\frac{\beta_2 e^{\frac{-\beta_2}{2}x_{(i)}^2}}{f(x_{(i)};\beta_1,\beta_2)}+\sum_{i=1}^{m} r_i\,\frac{e^{\frac{-\beta_2}{2}T_i^2}}{R(x;\beta_1,\beta_2)}$$

to both sides yields the following

$$\sum_{i=1}^{k}1+\sum_{i=1}^{m} r_i=n=\sum_{i=1}^{k}\frac{\beta_2 e^{\frac{-\beta_2}{2}x_{(i)}^2}}{f(x_{(i)};\beta_1,\beta_2)}+\sum_{i=1}^{m} r_i\,\frac{e^{\frac{-\beta_2}{2}T_i^2}}{R(x;\beta_1,\beta_2)} \qquad (11)$$

Since, from (9) we also have that

$$\sum_{i=1}^{k}\frac{\beta_1 x_{(i)} e^{\frac{-\beta_1}{2}x_{(i)}^2}}{f(x_{(i)};\beta_1,\beta_2)}+\sum_{i=1}^{m} r_i\,\frac{e^{\frac{-\beta_1}{2}T_i^2}}{R(x;\beta_1,\beta_2)}=\sum_{i=1}^{k}\frac{\beta_2 x_{(i)} e^{\frac{-\beta_2}{2}x_{(i)}^2}}{f(x_{(i)};\beta_1,\beta_2)}+\sum_{i=1}^{m} r_i\,\frac{e^{\frac{-\beta_2}{2}T_i^2}}{R(x;\beta_1,\beta_2)} \qquad (12)$$

We then obtain

$$\hat{p}_1=\frac{\hat{p}_1}{n}\sum_{i=1}^{k}\frac{\hat{\beta}_1 x_{(i)} e^{\frac{-\hat{\beta}_1}{2}x_{(i)}^2}}{f(x_{(i)};\hat{\beta}_1,\hat{\beta}_2)}+\sum_{i=1}^{m} r_i\,\frac{e^{\frac{-\hat{\beta}_1}{2}T_i^2}}{R(x;\hat{\beta}_1,\hat{\beta}_2)} \qquad (13)$$

From (10) we note that

$$\hat{\beta}_j=\left\{\sum_{i=1}^{k}\frac{x_{(i)} e^{\frac{-\beta_j}{2}x_{(i)}^2}}{f(x_{(i)};\hat{\beta}_1,\hat{\beta}_2)}+\sum_{i=1}^{m}\frac{r_i.T_i^2 e^{\frac{-\beta_j}{2}T_i^2}}{R(x;\hat{\beta}_1,\hat{\beta}_2)}\right\}\Big/\sum_{i=1}^{k}\left(x_{(i)}^3 e^{\frac{-\beta_j}{2}x_{(i)}^2}\Big/f(x_{(i)};\hat{\beta}_1,\hat{\beta}_2)\right) \qquad (14)$$

Then (13), (14) will determine a successive substitution iterative scheme.

If we examine the maximum likelihood estimating equations (9), (10), when we have no observed failures, i.e., $k=0$ we obtain

$$\sum_{i=1}^{m} r_i\,\frac{e^{\frac{-\beta_1}{2}T_i^2}-e^{\frac{-\beta_2}{2}T_i^2}}{R(x;\beta_1,\beta_2)}=0$$

$$\sum_{i=1}^{m} r_i p_j \frac{T_i^2 e^{\frac{-\beta_j}{2}T_i^2}}{2.R(x;\beta_1,\beta_2)} = 0$$

Since $\beta_1 > \beta_2$ and for $T_i > 0$ we have ($e^{\frac{-\beta_1}{2}T_i^2} - e^{\frac{-\beta_2}{2}T_i^2} > 0$), ($R(x;\beta_1,\beta_2) > 0$), and since not all $T_i's$ and $r_i's$ equal 0, the three equations are inconsistent. To exclude this trivial case, we take $\sum_{i=1}^{m} r_i < n$.

3. Asymptotic variance covariance matrix

The asymptotic Variance covariance matrix can be written as the inverse of the information matrix, where the negative of the information matrix is defined as:

$$\begin{bmatrix} -\mathrm{E}\left(\partial^2 \log L/\partial p_1^2\right) & -\mathrm{E}\left(\partial^2 \log L/\partial p\partial\beta_1\right) & -\mathrm{E}\left(\partial^2 \log L/\partial p\partial\beta_2\right) \\ -\mathrm{E}\left(\partial^2 \log L/\partial p\partial\beta_1\right) & -\mathrm{E}\left(\partial^2 \log L/\partial \beta_1^2\right) & -\mathrm{E}\left(\partial^2 \log L/\partial\beta_1\partial\beta_2\right) \\ -\mathrm{E}\left(\partial^2 \log L/\partial p\partial\beta_2\right) & -\mathrm{E}\left(\partial^2 \log L/\partial\beta_1\partial\beta_2\right) & -\mathrm{E}\left(\partial^2 \log L/\partial \beta_2^2\right) \end{bmatrix}$$

We determine the second partials by differentiating the first partials, equations (9) and (10), obtaining

$$\frac{\partial^2 \log L}{\partial p_1^2} = -\sum_{i=1}^{k} \frac{\left(\beta_1 x_{(i)} e^{\frac{-\beta_1}{2}x_{(i)}^2} - \beta_2 x_{(i)} e^{\frac{-\beta_2}{2}x_{(i)}^2}\right)^2}{\left(f(x_{(i)};\beta_1,\beta_2)\right)^2} - \sum_{i=1}^{m} r_i \frac{\left(e^{\frac{-\beta_1}{2}T_i^2} - e^{\frac{-\beta_2}{2}T_i^2}\right)^2}{\left(R(x;\beta_1,\beta_2)\right)^2} \tag{15}$$

$$\frac{\partial^2 \log L}{\partial \beta_j \partial p_1} = (-1)^{j-1} \sum_{i=1}^{k} \frac{\beta_k x_{(i)}\left(2x_{(i)} - \beta_j x_{(i)}^3\right) e^{\frac{-\beta_j}{2}x_{(i)}^2} e^{\frac{-\beta_k}{2}x_{(i)}^2}}{2\left[f(x_{(i)};\beta_1,\beta_2)\right]^2} + (-1)^{j-1} \sum_{i=1}^{m} r_i \frac{T_i^2 e^{\frac{-\beta_j}{2}T_i^2} e^{\frac{-\beta_k}{2}T_i^2}}{2\left[R(x;\beta_1,\beta_2)\right]^2},\ j \neq k \tag{16}$$

$$\frac{\partial^2 \log L}{\partial \beta_1 \partial \beta_2} = -\sum_{i=1}^{k} p_1 p_2 \frac{e^{\frac{-\beta_1}{2}x_{(i)}^2}\left(2x_{(i)} - \beta_1 x_{(i)}^3\right)\left(2x_{(i)} - \beta_2 x_{(i)}^3\right) e^{\frac{-\beta_2}{2}x_{(i)}^2}}{4\left[f(x_{(i)};\beta_1,\beta_2)\right]^2} - \sum_{i=1}^{m} p_1 p_2 r_i \frac{T_i^4 e^{\frac{-\beta_1}{2}T_i^2} e^{\frac{-\beta_2}{2}T_i^2}}{4\left[R(x;\beta_1,\beta_2)\right]^2} \tag{17}$$

$$\frac{\partial^2 \log L}{\partial \beta_j^2} = \sum_{i=1}^{k}\left[p_j \frac{x_{(i)}^2\left(\beta_j x_{(i)}^3 - 4x_{(i)}\right)e^{\frac{-\beta_j}{2}x_{(i)}^2}}{4f(x_{(i)};\beta_1,\beta_2)} - \frac{p_j^2 . e^{\frac{-\beta_j}{2}x_{(i)}^2}\left(2x_{(i)} - \beta_j x_{(i)}^3\right)^2}{4\left[f(x_{(i)};\beta_1,\beta_2)\right]^2}\right]$$
$$+\sum_{i=1}^{m}\left[p_j r_i \frac{T_i^4 e^{\frac{-\beta_j}{2}T_i^2}}{4R(x;\beta_1,\beta_2)} - r_i \frac{\left(p_j T_i\, e^{\frac{-\beta_j}{2}T_i^2}\right)^2}{4\left[R(x;\beta_1,\beta_2)\right]^2}\right] \tag{18}$$

The expected values of the above second partial derivatives will be calculated by transforming the original integrals into a form more suitable for numerical integration. Hill (1963) is also concerned with this problem but deals with less general case when $\beta_1,\ \beta_2$ are known and only the mixing proportion is estimated.

$$-\mathrm{E}\left(\frac{\partial^2 \log L}{\partial p_1^2}\right) = \frac{k}{p_1 p_2}\left[1 - S_1\right] - \sum_{i=1}^{m} r_i \frac{\left(e^{\frac{-\beta_1}{2}T_i^2} - e^{\frac{-\beta_2}{2}T_i^2}\right)^2}{\left(R(x;\beta_1,\beta_2)\right)^2} \tag{19}$$

$$-\mathrm{E}\left(\frac{\partial^2 \log L}{\partial \beta_1 \partial p_1}\right) = \frac{k}{2\beta_1}\left(\beta_1 S_2 - 2S_1\right) + \sum_{i=1}^{m} r_i \frac{T_i^2 e^{\frac{-\beta_1}{2}T_i^2} e^{\frac{-\beta_2}{2}T_i^2}}{2\left[R(x;\beta_1,\beta_2)\right]^2} \tag{20}$$

$$-\mathrm{E}\left(\frac{\partial^2 \log L}{\partial \beta_2 \partial p_1}\right) = \frac{-k}{2\beta_2}\left(\beta_2 S_2 - 2S_1\right) - \sum_{i=1}^{m} r_i \frac{T_i^2 e^{\frac{-\beta_1}{2}T_i^2} e^{\frac{-\beta_2}{2}T_i^2}}{2\left[R(x;\beta_1,\beta_2)\right]^2} \tag{21}$$

$$-\mathrm{E}\left(\frac{\partial^2 \log L}{\partial \beta_1 \partial \beta_2}\right) = \frac{-k.p_1 p_2}{4\beta_1.\beta_2}\left[S_3\left(4\beta_2 + 2\beta_1\right) - \beta_1.\beta_2 S_2 - 4S_1\right] + \sum_{i=1}^{m} p_1 p_2 r_i \frac{T_i^4 e^{\frac{-\beta_1}{2}T_i^2} e^{\frac{-\beta_2}{2}T_i^2}}{4\left[R(x;\beta_1,\beta_2)\right]^2} \tag{22}$$

$$-\mathrm{E}\left(\frac{\partial^2 \log L}{\partial \beta_1^2}\right) = \frac{k.p_1}{4}\left[p_1 S_5 - S_4\right] + \sum_{i=1}^{m}\left[p_1 r_i \frac{T_i^4 e^{\frac{-\beta_1}{2}T_i^2}}{4R(x;\beta_1,\beta_2)} - r_i \frac{\left(p_1 T_i\, e^{\frac{-\beta_1}{2}T_i^2}\right)^2}{4\left[R(x;\beta_1,\beta_2)\right]^2}\right] \tag{23}$$

$$-\mathrm{E}\left(\frac{\partial^2 \log L}{\partial \beta_2^2}\right) = \frac{k.p_2}{4}\left[p_2 S_7 - S_6\right] + \sum_{i=1}^{m}\left[p_2 r_i \frac{T_i^4 e^{\frac{-\beta_2}{2}T_i^2}}{4R(x;\beta_1,\beta_2)} - r_i \frac{\left(p_2 T_i\, e^{\frac{-\beta_2}{2}T_i^2}\right)^2}{4\left[R(x;\beta_1,\beta_2)\right]^2}\right] \tag{24}$$

We explained how to find these values $S_1,\ldots,S_7$ in the Appendix. Note that, the expected values of the first summations are variable quantities and the second summations equal zero.

4. Numerical examples

In this section, we present results of some numerical experiments to compare the estimators performance of the different sample schemes proposed in the previous sections.

4.1. Illustrative examples: Using "MATHCAD" (13), a sample of size 600 was generated random mixture of Rayleigh numbers with parameters $\beta_1 = 0.6$, $\beta_2 = 4$ and $p_1 = 0.5$ based on progressive type I censored which occurs at five stages $m = 5$. Assume that at time $T_1 = 0.6$, twelve units selected at random from the survivors, were censored (i.e. removed from the test).
At $T_2 = 2$, ten additional randomly selected survivors were removed. Three additional randomly select survivors were removed at $T_3 = 3.4$. At time $T_4 = 4.6$, another two units selected at random from the survivors, and the test was terminated at $T_5 = 5.2$ with three survivors.

Table 1 shows the maximum likelihood estimators for the unknown parameters of the mixed Rayleigh model under progressive type I censored, as a special case, we show the maximum likelihood estimators for the unknown parameters in both of type I censored when $T_5 = T = 5.2$, thus, we have $k = 423$ failures, and $n - k = 123$ survivors to be removed from the test and complete sample; using these data, we have

Table 1

Estimators	Progressive Type I Censored	Type I Censored	Complete sample
$\hat{p}_1$	0.5431	0.512	0.497
$\hat{\beta}_1$	0.872	0.726	0.436
$\hat{\beta}_2$	4.816	5.034	3.924

- Comparison between estimators of different sample schemes

Again, **Table 2** shows asymptotic variance covariance matrix of the maximum likelihood estimators p_1, β_1 and β_2 were calculated as described in section (3) and are given as

Table 2

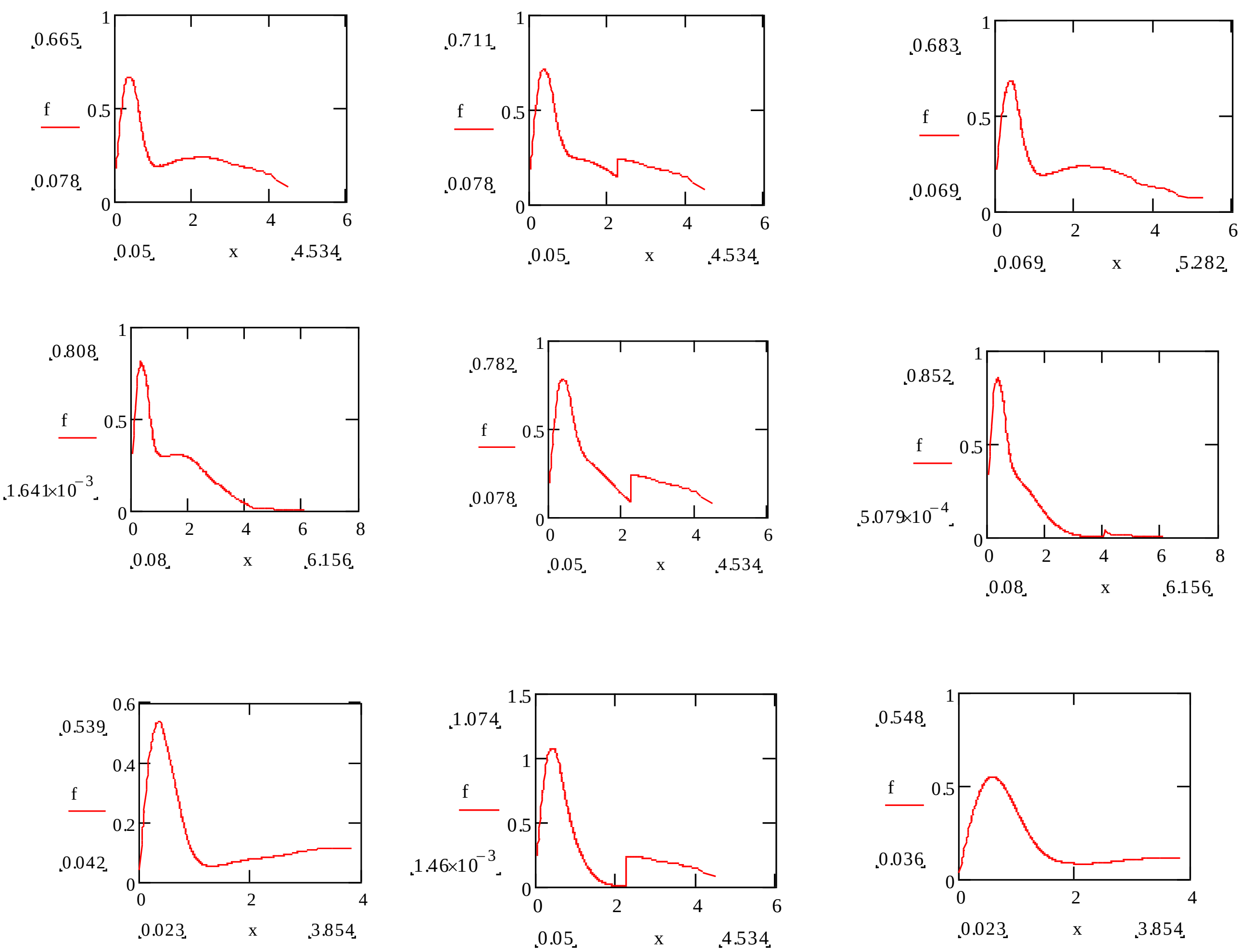
schemes	Asymptotic variance covariance matrix					
	$V(p_1)$	$V(\beta_1)$	$V(\beta_2)$	$Cov(p_1,\beta_1)$	$Cov(p_1,\beta_2)$	$Cov(\beta_1,\beta_2)$
Progressive Type I Censored	0.02175	0.0073	0.0098	-0.0724	-0.0088	0.0046
Type I Censored	0.00841	0.0051	0.0091	-0.0681	-0.0064	0.0043
Complete sample	0.0007	0.0004	0.0064	-0.0233	-0.0041	0.0032

Comparison between asymptotic variance covariance matrix of different sample schemes

When $p_1 = 0.5$, the next figure shows comparisons on the graphs of Mixed Raleigh Distribution between different sample schemes (complete sample, progressive Type I censored and Type I censored).

Figure

Progressive Type I Censored **Type I Censored** **Complete sample**



For each row the same values from the scale parameters β_1 and β_2 as follow: $\beta_1 = 0.6, \beta_2 = 4, \beta_1 = 1, \beta_2 = 5$ and $\beta_1 = 3, \beta_2 = 0.2$ respectively.

4.2. Simulation results: In the following, the maximum likelihood estimators under different samples schemes are compared via Monte Carlo simulation.

Using the approximate maximum likelihood estimators as the starting value, the maximum likelihood estimators are obtained by solving the nonlinear Equations (2.6) and (2.7) using the Newton's method. We generate a mixed Rayleigh distribution in type I progressive censored sample with $\beta_1 = 3, \beta_2 = 0.2$ and $p_1 = 0.5$ from a sample of sizes 15, 20 and 30. The simulations are carried out for different stages m .

Tables 3

n	m	Progressive Type I Censored			Type I Censored			Complete sample		
		$\hat{p}_1$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{p}_1$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{p}_1$	$\hat{\beta}_1$	$\hat{\beta}_2$
15	2	0.393	3.341	0.281	0.437	3.246	0.225	0.495	3.101	0.194
	3	0.389	3.499	0.211	0.435	3.424	0.209	0.482	3.370	0.186
	5	0.354	3.577	0.215	0.421	3.490	0.258	0.459	3.414	0.174
20	3	0.471	3.082	0.277	0.477	3.064	0.276	0.486	3.022	0.209
	4	0.469	3.248	0.253	0.469	3.196	0.248	0.483	3.162	0.204
	6	0.457	3.410	0.245	0.479	3.392	0.211	0.476	3.319	0.201
30	3	0.540	3.111	0.216	0.532	3.083	0.214	0.505	3.010	0.204
	4	0.556	3.199	0.269	0.541	3.143	0.234	0.511	3.102	0.207
	5	0.591	3.339	0.311	0.555	3.318	0.266	0.546	3.287	0.250
	8	0.594	3.689	0.337	0.568	3.639	0.333	0.564	3.590	0.316

Tables 3 provide the maximum likelihood estimators with different Sample schemes as mentioned above. For all sample sizes, the maximum likelihood estimators under different samples schemes are almost the same values, but the complete samples estimators are the best for ever; however, it is seen that the progressive type I censored is better than the other samples, where it reduces the consumption of the sample size.

5. Conclusions

In this study maximum likelihood estimation of the parameters of a mixture of two Rayleigh distributions, $f(x;\beta_1,\beta_2) = p_1\beta_1.x.e^{-\frac{\beta_1}{2}x^2} + p_2\beta_2.x.e^{-\frac{\beta_2}{2}x^2}$ has been examined in detail. The asymptotic variance covariance matrix was obtained by taking the inverse of the information matrix, which required numerical integration (Appendix). Also, we have considered the estimation procedure and asymptotic variance covariance matrix under various forms of censoring.

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Appendix

From (3.1) we have

$$-\mathrm{E}\left(\frac{\partial^2 \log L}{\partial p_1^2}\right)=k\int_0^\infty \frac{\left(\beta_1 x_{(i)} e^{\frac{-\beta_1}{2}x_{(i)}^2}-\beta_2 x_{(i)} e^{\frac{-\beta_2}{2}x_{(i)}^2}\right)^2}{f(x_{(i)};\beta_1,\beta_2)}dx-\sum_{i=1}^m r_i \frac{\left(e^{\frac{-\beta_1}{2}T_i^2}-e^{\frac{-\beta_2}{2}T_i^2}\right)^2}{\left(R(x;\beta_1,\beta_2)\right)^2}$$

but

$$\int_0^\infty \frac{\left(\beta_1 x_{(i)} e^{\frac{-\beta_1}{2}x_{(i)}^2}-\beta_2 x_{(i)} e^{\frac{-\beta_2}{2}x_{(i)}^2}\right)^2}{f(x_{(i)};\beta_1,\beta_2)}dx=$$

$$=\frac{-1}{p_1 p_2}\int_0^\infty \left\{\frac{\left[f(x_{(i)};\beta_1,\beta_2)-\beta_1 x_{(i)} e^{\frac{-\beta_1}{2}x_{(i)}^2}\right]\left[f(x_{(i)};\beta_1,\beta_2)-\beta_2 x_{(i)} e^{\frac{-\beta_2}{2}x_{(i)}^2}\right]}{f(x_{(i)};\beta_1,\beta_2)}\right\}$$

$$=\frac{1}{p_1 p_2}\left\{1-\int_0^\infty \beta_1\beta_2 x_{(i)}^2 e^{\frac{-\beta_1}{2}x_{(i)}^2} e^{\frac{-\beta_2}{2}x_{(i)}^2} / f(x_{(i)};\beta_1,\beta_2)dx\right\}$$

If we let $S_1=\int_0^\infty \beta_1\beta_2 x_{(i)}^2 e^{\frac{-\beta_1}{2}x_{(i)}^2} e^{\frac{-\beta_2}{2}x_{(i)}^2} / f(x_{(i)};\beta_1,\beta_2)dx$, we obtain

$$-\mathrm{E}\left(\frac{\partial^2 \log L}{\partial p_1^2}\right)=\frac{k}{p_1 p_2}\left[1-S_1\right]-\sum_{i=1}^m r_i \frac{\left(e^{\frac{-\beta_1}{2}T_i^2}-e^{\frac{-\beta_2}{2}T_i^2}\right)^2}{\left(R(x;\beta_1,\beta_2)\right)^2}$$

Note that in the appendix we provide a means of calculating S_1 and the ensuing integrals $S_2 \ldots S_7$.

By using the method previously

$$S_2=\int_0^\infty \beta_1\beta_2 x_{(i)}^4 e^{\frac{-\beta_1}{2}x_{(i)}^2} e^{\frac{-\beta_2}{2}x_{(i)}^2} / f(x_{(i)};\beta_1,\beta_2)dx \ ,$$

$$S_3=\int_0^\infty \beta_1\beta_2 x_{(i)}^3 e^{\frac{-\beta_1}{2}x_{(i)}^2} e^{\frac{-\beta_2}{2}x_{(i)}^2} / f(x_{(i)};\beta_1,\beta_2)dx,$$

$$S_4=\int_0^\infty x_{(i)}^2\left[\beta_1 x_{(i)}^3-4x_{(i)}\right]e^{\frac{-\beta_1}{2}x_{(i)}^2} / f(x_{(i)};\beta_1,\beta_2)dx,$$

$$S_5=\int_0^\infty \left[2x_{(i)}-\beta_1 x_{(i)}^3\right]^2 e^{\frac{-\beta_1}{2}x_{(i)}^2} / f(x_{(i)};\beta_1,\beta_2)dx,$$

$$S_6 = \int_0^\infty x_{(i)}^2 \left[\beta_2 x_{(i)}^3 - 4x_{(i)}\right] e^{\frac{-\beta_2}{2} x_{(i)}^2} / f(x_{(i)}; \beta_1, \beta_2) dx,$$

and

$$S_7 = \int_0^\infty \left[2x_{(i)} - \beta_2 x_{(i)}^3\right]^2 e^{\frac{-\beta_2}{2} x_{(i)}^2} / f(x_{(i)}; \beta_1, \beta_2) dx.$$