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ADAPTED FUZZY CONTROLLER FOR ASTRONOMICAL TELESCOPE TRACKING

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Abstract. This paper presents a novel application of fuzzy logic (FL) controller driven by an adaptive fuzzy set (AFS) for position tracking of the telescope driven by electric motor. Also, the proposed FL controller, driven by AFS, is compared with a classical FL control, driven by a static fuzzy set (SFS). Both FL controllers algorithm use the position error and its rate of change as an input vector. The mathematical model of the telescope driven by electric motor is highly nonlinear differential equations. Therefore the use of the artificial intelligent controller, such as FL is much better than the conventional controller, to cover a wide range of operating conditions. So, the output of FL control is utilized to force the electric drives, of the telescope, to satisfy a perfect matching of the predefined desired position of the telescope arms. Both of FL controllers, using AFS and SFS, are simulated and tested when the system is subjected to a step change in reference value. In addition, these simulation results are compared with the conventional Proportional-Derivative (PD) controller, driven by fixed gain. The proposed FL, using an adaptive fuzzy set, improve the dynamic response of the overall system by improving the damping coefficient and decreasing the rise time and settling time compared with other two controllers.

Keywords: adapted and static fuzzy sets, fuzzy logic controller.

1. Introduction

The problem of tracking a given trajectory is common in many industrial drives plant. In motor drive applications, this may require that the motor follow its predetermined position or speed trajectory during starting, speed reference change and braking. Also, the reference tracking should take place without causing excessive stresses to the entire system hardware nor excessive inrush current into the electrical motor driver. Therefore in order to achieve this requirement, the control strategy must be adaptive, robust, accurate, and simple to implement Attia et al. (2001).

The mathematical equations of the astronomical telescope driven by electrical motor are described through highly nonlinear-coupled differential equations (Attia, 1997). These equations contain a varying inertia term, a centrifugal and coriolis term, and gravity term (Attia, 1997). Meanwhile, the gravity term tends to zero for a well-balanced telescope. The system nonlinearity imposes difficulty to design an accurate conventional controller to cover wide range of operating points in nonlinear-coupled differential equations. Introducing compensator, based on state feedback linearization, is aimed at decoupling system, hence improving its dynamic response (Attia, 2004).

Fuzzy logic controller has been applied successfully in several applications such as control of astronomical telescope (Attia et al. 2001; Soliman et al. 1998) electrical machines control F. Franklin et al. (1995), and fault diagnosis (Kazmierkowski and Malesani, 1998).

Adaptive fuzzy controllers provide a mean of continuously adapting the fuzzy rules to match desired performance criteria. Its typical application area is the control of time varying and/or nonlinear plants. Fuzzy controllers may be either static or dynamic. The static fuzzy rules are usually based on operator experience as fuzzy logic can easily encode linguistic information (Attia, et al. 2001; Franklin, et al. 1995). This is the main advantage of fuzzy logic controller over neural networks. In the adaptive case, the linguistic information captured from operator experience can be used to initialize the fuzzy rules. This helps to reduce the training's number of iterations Nurnberger et al. (1999).

In this paper, two fuzzy controllers are proposed to control the electrical motor driving astronomical telescope. The first one has static fuzzy rules which are based on human experience. It is similar to the one proposed in previous work Attia et al. (2001). However, as the operating conditions of the telescope changes, it is expected that the controller parameters require fine tuning. The second controller uses an adaptation scheme which dynamically varies the rules of the fuzzy set to achieve a better dynamic performance. Both static and adaptive fuzzy logic controllers performance are compared to that for the conventional PD controller.

2. Mathematical model of 14″ Celestron telescope

The 14″ Celestron telescope is a fork-mounted Schmidt Cassegrain (Celestron, 1992), as shown in Figure 1. It includes an optical tube assembly, an electric clock drive with a worm gear drive and a giant 2″ star diagonal. In addition, there is a 14″ visual back, a 10×40 finder scope, setting circles, two counter weights bar assemblies, a lens cap, carrying cases, and permanent magnet dc motor drives in the Right Ascension (RA) and Declination (DEC) axis to move the telescope on both sides. The dynamic equations of the telescope are a set of highly nonlinear-coupled differential equations containing a varying inertia term, a centrifugal and coriolis term, a frictional term, and a gravity term equal to zero. These equations are plugged by substantial requirements for computation and the theory underlying their solution is incomplete. The dynamic equations of the telescope are given by Eqs. (1) and (2).

$$M(\theta)\ddot{\theta} + C(\theta, \dot{\theta}) + G(\theta) = \tau, \tag{1}$$

$$M(\theta)\ddot{\theta} + N(\theta, \dot{\theta}) + \tau_d = \tau, \tag{2}$$

where: θ, $\dot{\theta}$, and $\ddot{\theta}$ are the joint angular position, velocity and acceleration vectors respectively. τ_d is a constant disturbance torque, which represents the unknown

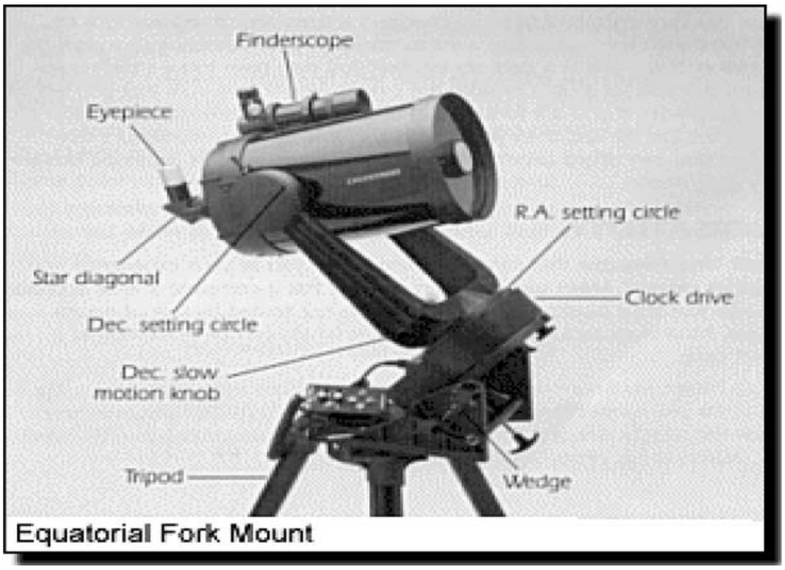

Figure 1. 14″ Celestron telescope.

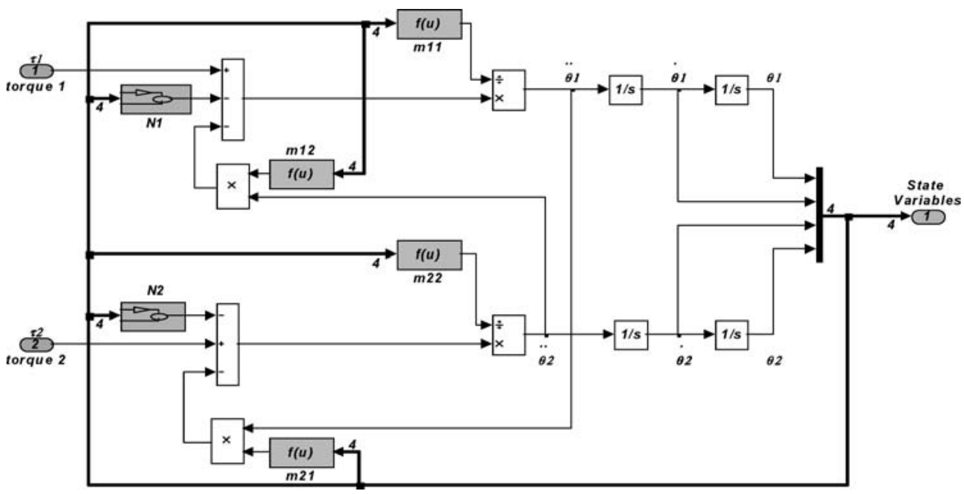

Figure 2. Simulink block diagram for the Celestron Telescope Model.

dynamics, e.g. friction. Also, each angular position contains the following variables:

$$\theta = [\theta_1\ \theta_2]^T, \dot{\theta} = [\dot{\theta}_1\ \dot{\theta}_2]^T,\ \ddot{\theta} = [\ddot{\theta}_1\ \ddot{\theta}_2]^T, \quad N(\theta, \dot{\theta}) = C(\theta, \dot{\theta}) + G(\theta).$$

where C is a vector of centrifugal and coriolis terms, G is a vector of gravity terms as defined in Appendix. Figure 2 shows the Simulink block diagram for the Celestron telescope model.

For a coupled system like a telescope model, the input torques τ_1 and τ_2 control the outputs θ_1 and θ_2. It is required to decouple this model by introducing a compensator. Consequently a decoupled system can be considered as consisting of a set of independent single-input single-output systems. The compensator with the model of the telescope results into a linear system, which enables a linear controller such as a PD controller to be used for control. The overall input for nonlinear model of the telescope consists of a compensator plus two PD feedback controllers running in parallel. The controller is consisting of a state feedback compensator and a linear controller processes with state vector $[\theta_1 \;\; \dot{\theta}_1 \;\; \dot{\theta}_2 \;\; \theta_1]$ (see ref. (Attia, 1997)).

3. Computed-torque controllers

The computed-torque control with an auxiliary control signal PD feedback is selected for each actuator input, based on the local measurements of position errors, and the joint velocity for each arm Lewis et al. (1993). In fact, there is no tachometer to measure joint velocities. These velocities are estimated from angle measurements, as shown in Figure 3. The output of the PD controller is described by Equation (3) as shown in Figures (3) and (4) Slotine et al. (1991).

$$s = -k_p e_\theta - k_v \dot{e}_\theta \tag{3}$$

The feedback linearizing transformation becomes as in Equation (4):

$$\tau = M(\theta)(\ddot{\theta}_d(t) - S) + N(\theta, \dot{\theta}). \tag{4}$$

So, the overall input to the nonlinear model becomes as in Equation (5):

$$\tau = M(\theta)(k_p e_\theta + k_v \dot{e}_\theta) + N(\theta, \dot{\theta}). \tag{5}$$

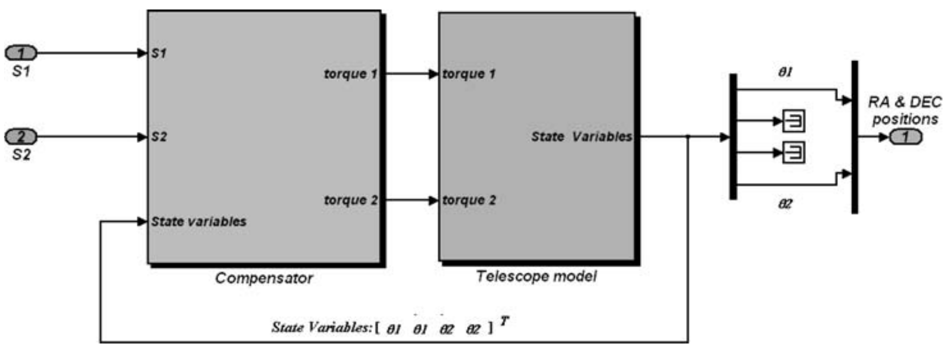

Figure 3. Linear telescope model.

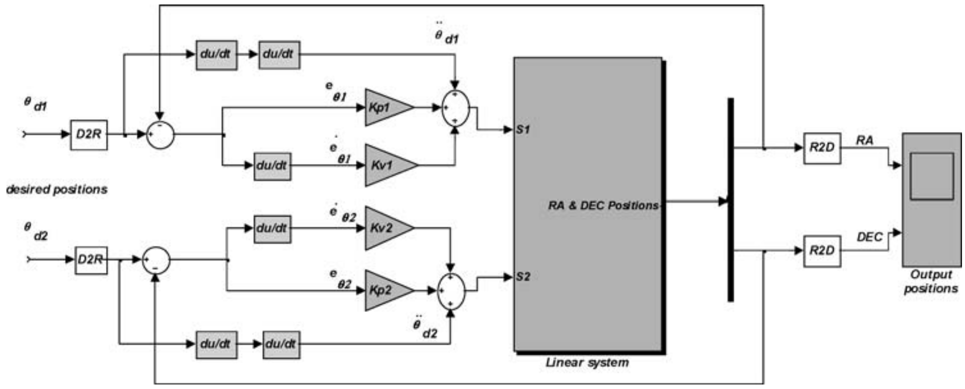

Figure 4. PD computed-torque controllers for the telescope model.

where k_p, k_v are the proportional and the derivative gains of the PD controllers. e_θ is the difference between the desired position angle and the actual position angle, i.e. $e_\theta = \theta_d - \theta$, of the telescope in a certain direction for Right ascension or declination positions.

Then, the optimal gains of the PD controllers $[k_{p1}, k_{v1}, k_{p2}$, and $k_{v2}]^T$, as shown in Figure 4, are determined by the Ziegler-Nichols rule using nonlinear block-set in MATLAB package. Also, these controller gains could be designed based on genetic algorithms as in Attia et al. (2001, 2004).

Assuming the initial angles of the telescope are $\theta^\circ = [0^\circ\ 0^\circ]^T$, and the desired step angles are $\theta_d = [45^\circ\ 30^\circ]^T$. The transient positions are translated into degrees, while all the control computations and gains are used in radians.

4. Static fuzzy logic controller (SFLC)

Conventional control based on modern scientific analysis determines the control effort in relation to a number of data inputs using a set of equations to express the control process. Expressing human experience in the form of a mathematical formula is a very difficult task, perhaps an impossible one. Fuzzy logic has provided the simple tool to interpret this experience to reality. According to Lee, a fuzzy logic controller can be simply represented in four parts Lee et al. (1990) as shown in Figure 5:

- **Fuzzification interface** is responsible first for reading (measuring, and scaling) the fuzzy control variables, which are the position deviation from the desired value (e_θ), and its rate of change ($\dot{e}_\theta$). Then interpreting the measured numeric values, (e_θ) and ($\dot{e}_\theta$) to corresponding linguistic variables with appropriate membership value as shown in Figure 5.
- **Knowledge base representation** that provides the definitions of the fuzzy membership functions defined for every fuzzy control variable; five triangular fuzzy

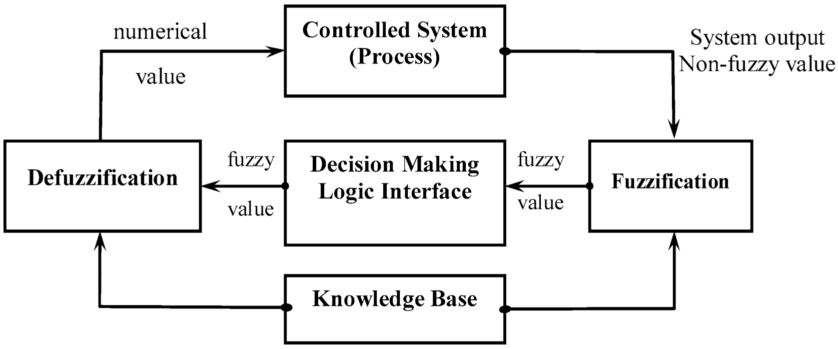

Figure 5. The basic configuration of a fuzzy logic controller.

sets are selected in this paper, and the necessary fuzzy rules are constructed, which specify the control goals using linguistic fuzzy terms.

- **The inference system**, the heart of the controller, provides approximate reasoning based on the knowledge base. It should be capable of simulating human decision making and inferring the control actions based on fuzzy logic.
- **The defuzzification interface** translates fuzzy control action to nonfuzzy control action, i.e. numerical values using a digital Center of Area (COA) as expressed in the following equation (6).

$$COA = \frac{\sum_{i=1}^{N} \mu_i u_i}{\sum_{i=1}^{N} \mu_i}. \tag{6}$$

where μ_i is the centroid of the i^{th} membership function, u_i is a constant, which determines the spread of the i^{th} membership function, N is the number of linguistic variables.

The first step in fuzzy controller design was the selection of input and output variables. The designated input variables were (position error, e_θ) and (velocity of the telescope link, $\dot{e}_\theta$). Each variable is accompanied with a set of membership functions as shown in Figures (6-a, 6-b), which represent the normalized triangular membership functions as expressed in linguistic variables for RA, and it can be the same for DEC Soliman et al. (1998); Attia et al. (2001). The linguistic variables use the symbol P for positive, N for negative, S for small, M for medium and Z for zero. The fuzzy set used for the output is shown in Figure 7. The generation of the output fuzzy set using correlation minimum encoding is done being the method of center of area as expressed in equation (6).

Table I shows the look-up of rules for input variables $e_{\theta 1}$, and $\dot{e}_{\theta 1}$. Figure 8 shows rules surface viewer of RA SFLC controller for the inputs and output torque of RA direction.

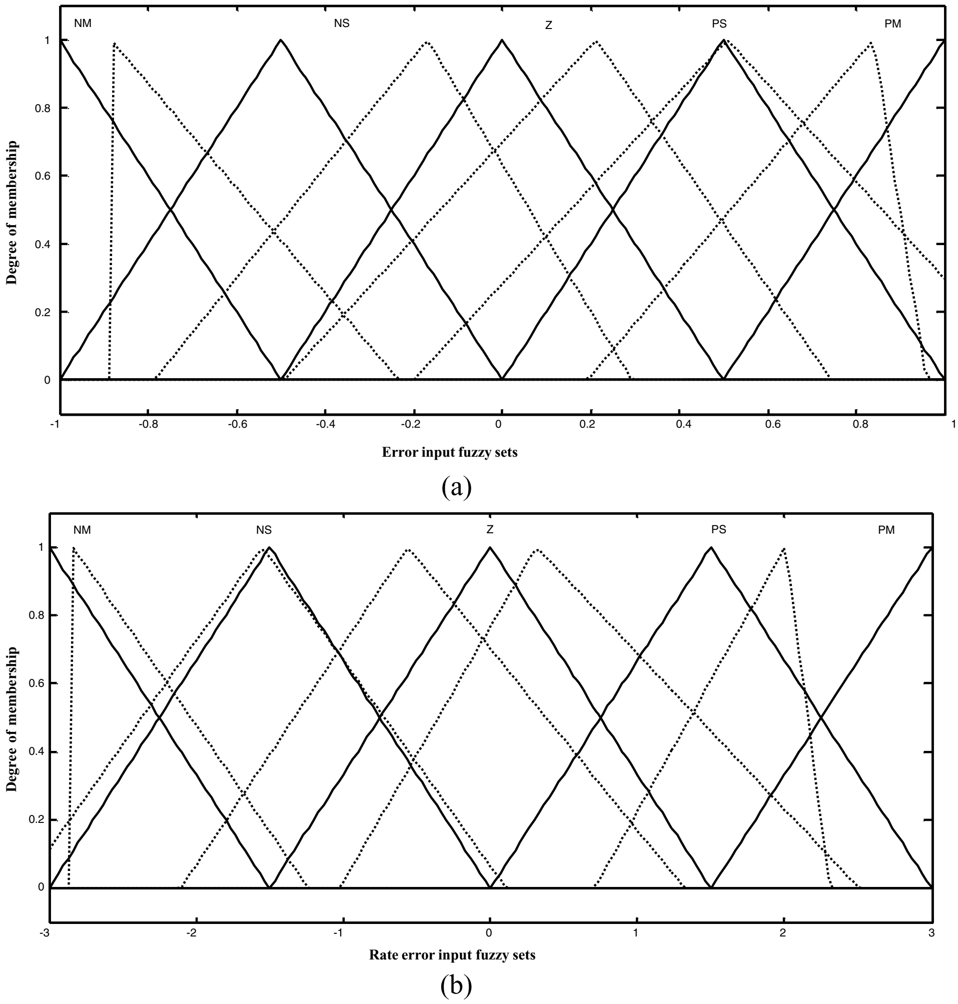

*Figure 6*a,b. Solid line: Optimized *MFs* before learning. Dashed line: Normalized *MFs* after learning.

During the actual operation, the computer would read in the Celestron position data and compute position error and velocity. The fuzzy controller fuzzified the input quantities through algorithms that operated on the input data as specified by the membership functions. Next, the fuzzified input quantities passed through a series of *If* and *Then* decision rules that formed the main body of the fuzzy controller assessed the current state of the telescope and determined which control action was most appropriate. Defuzzification was applied using the output variable and the control action was selected. The widely used center of area strategy generates the center of gravity of the possibility distribution of a control action. The control action is the torques of the motor drive of both the two arms of the Celestron telescope RA and

TABLE I
Look-up table of $e_{\theta 1}$, and $\dot{e}_{\theta 1}$

	$e_{\theta 1}$				
$\dot{e}_{\theta_1}$	NM	NS	Z	PS	PM
NM	NM	NM	NM	NS	Z
NS	NM	NM	NS	Z	PS
Z	NM	NS	Z	PS	PM
PS	NS	Z	PS	PM	PM
PM	Z	PS	PM	PM	PM

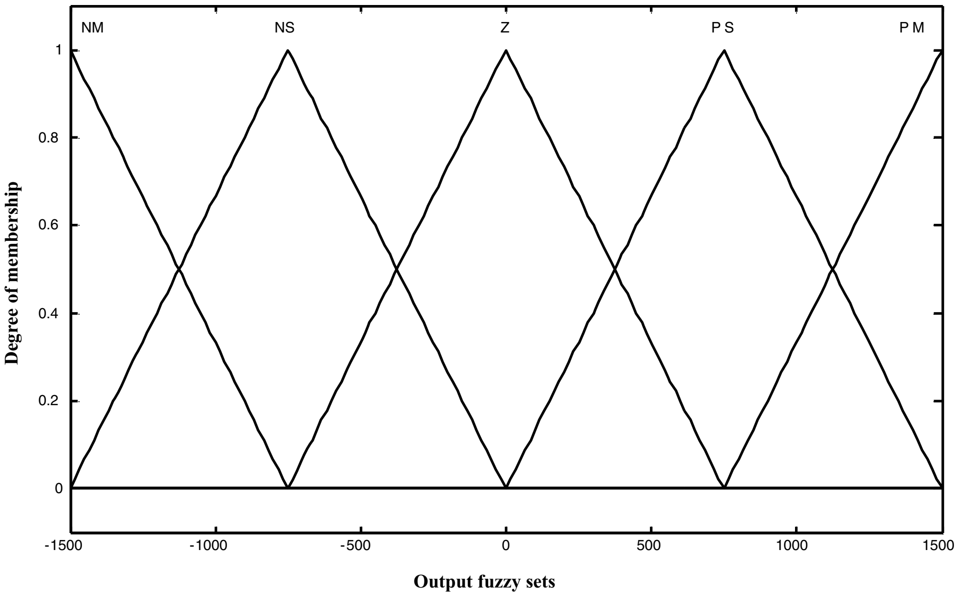

Figure 7. Normalized output membership functions for Torque in RA.

DEC directions. The implementation and simulation results shown in Figures (10, 11) are generated using MATLAB compiler.

5. Adapted fuzzy logic controller

The adaptive fuzzy logic controller (AFLC), using adaptive fuzzy set, has the same inputs and output as the static case SFLC. A full rule base (25 rules) is also defined. The rules have the general form:

If e_θ is NS and $\dot{e}_\theta$ is Z then τ is NS.

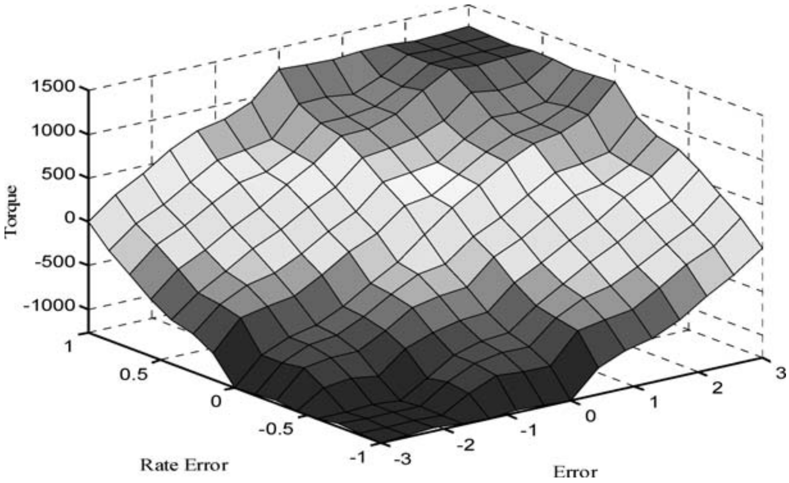

Figure 8. Rules Surface Viewer of RA SFLC Controller.

Where the membership functions (mf_i) is defined as follows:

$mf_j \in \{NM, NS, Z, PS\ and\ PM\}$ as in the static fuzzy case. However, the output space has 5 different fuzzy sets. To accommodate for the change in operating conditions, the adaptation algorithm changes the parameters of the input fuzzy sets.

The algorithm presented in this section is designated to optimize a rule base of the fuzzy controller by shifting and/or modifying the support of the input fuzzy sets. They do not modify the rules or the structure of the fuzzy controller. In the following, we assume that the fuzzy sets $\mu_j^{(i)}$ representing the linguistic triangular membership functions of the input variables respectively are defined as follows:

$$\mu_j^{(i)}(x; a, b, c) \overset{def}{=} \begin{cases} 0 & x \le a \\ \dfrac{x - a_j^{(i)}}{b_j^{(i)} - a_j^{(i)}} & a \le x \le b \\ \dfrac{c_j^{(i)} - x}{c_j^{(i)} - b_j^{(i)}} & b \le x \le c \\ 0 & c \le x \end{cases}$$

Where $a_j^{(i)}, b_j^{(i)}, c_j^{(i)} \in \Re, a_j^{(i)} \le b_j^{(i)} \le c_j^{(i)}$ Nurnberger et al. (1999), $a, b,$ and c are constants and $\Re$ is the real integer number set. This means that $\mu_j^{(i)}(a_j^{(i)}) = 0$, $\mu_j^{(i)}(b_j^{(i)}) = 1$, and $\mu_j^{(i)}(c_j^{(i)}) = 0$. Figure 9 shows the triangular membership functions will be used as symmetric fuzzy sets in the consequents and antecedents.

The updated parameters of the membership functions (*MFs*) of the fuzzy sets for the input variables using the on-line back-propagation (BP) algorithm could be

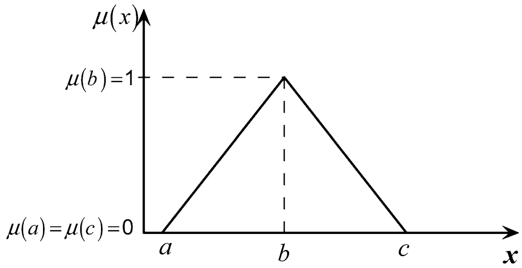

Figure 9. Triangular membership function (*MF*).

expressed in a simple form as follows:

$$\phi\{k+1\} = \phi\{k\} + \alpha \times \Delta\phi\{k\},$$

where α represents the learning rate, and ϕ represents the parameters $a_j^{(i)}$, $b_j^{(i)}$, $c_j^{(i)}$ $\in \Re$, $a_j^{(i)} \leq b_j^{(i)} \leq c_j^{(i)}$ of the membership function of the fuzzy sets for the input variables. $\Delta\phi\{k\}$ is the change of these parameters based on the performance of the system under study. While the parameters of the membership functions of the output fuzzy sets for the output torque are fixed Nurnberger et al. (1999). Figure 6 shows the normalized *MFs* of the input fuzzy sets before and after learning.

6. Simulation results

The nonlinear differential equations, described the system under study, is solved using the Runge-Kutta fifth order method using MATLAB Simulink package. The integration step value is automatically varied in this package. The tolerance was set at 0.001, the minimum and maximum step size was adjusted automatically. The sampling time was 0.01 seconds. Several tests are carried out to validate the efficiency of the proposed control schemes. The PD controller gains of the RA and the DEC direction arms are selected to be $Kp_1 = 220$, $Kv_1 = 90$, $Kp_2 = 200$ and $Kv_2 = 85$, respectively. These PD controller gains, values of Kp_1, Kp_2, Kv_1, and Kv_2, are tested in reference (Attia, 2004). The tuning strategy goal for the AFLC is to achieve a fast dynamic response with no overshoot, and negligible steady state error. The complexity of the FL controller is reduced due to reducing the number of rules and adapting the membership functions parameters. Figures (6-a) and (6-b) show the normalized *MFs* before and after training using BP technique for the input variables of the fuzzy controller. The dynamic responses of the position using a PD, SFLC and AFLC controllers for RA, and DEC movement arms are shown in Figure (10) and Figure (11) respectively. The dynamic response when using AFLC is superior in comparison with the other two controller regarding the rising time, settling time and damping coefficient of the overall system. These dynamic responses are carried

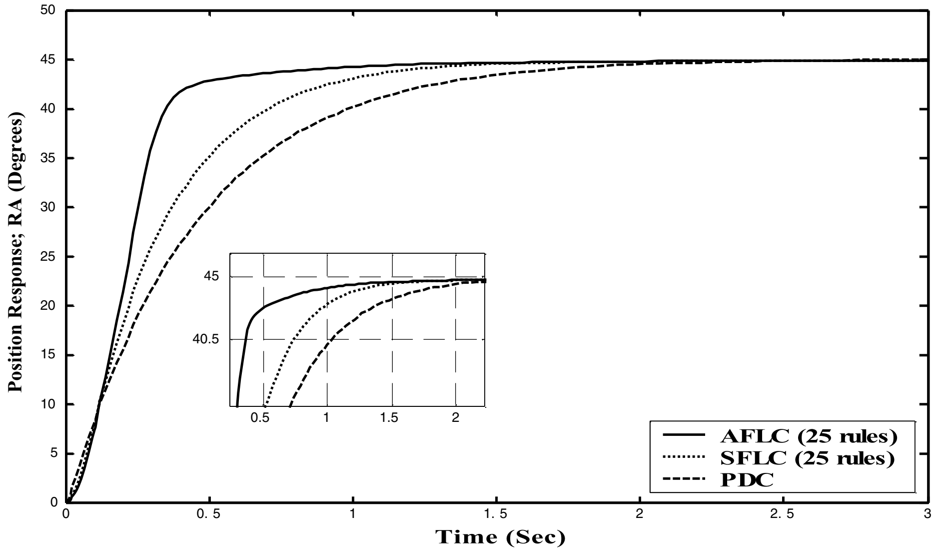

Figure 10. RA position response based on PDC, SFLC and AFLC Controllers.

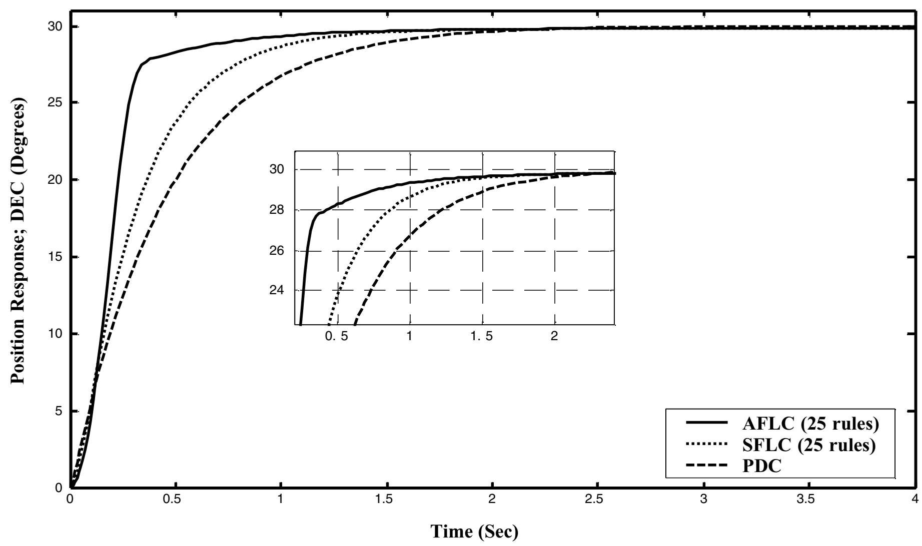

Figure 11. DEC position response based on PDC, SFLC and AFLC Controllers.

out when the desired values of the telescope positions for RA and DEC arms are set such that $\theta_d = \left[\, 45^\circ \; 30^\circ \,\right]^T$.

The variations of the velocity for RA and DEC arms are shown in Figures (12) and (13). The velocity values become zero when the system reaches to the

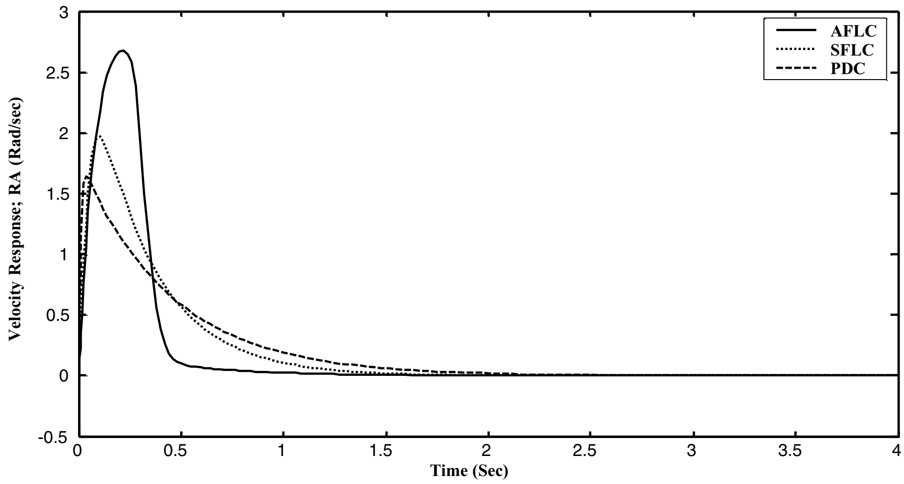

Figure 12. RA Velocity response based on PDC, SFLC and AFLC Controllers.

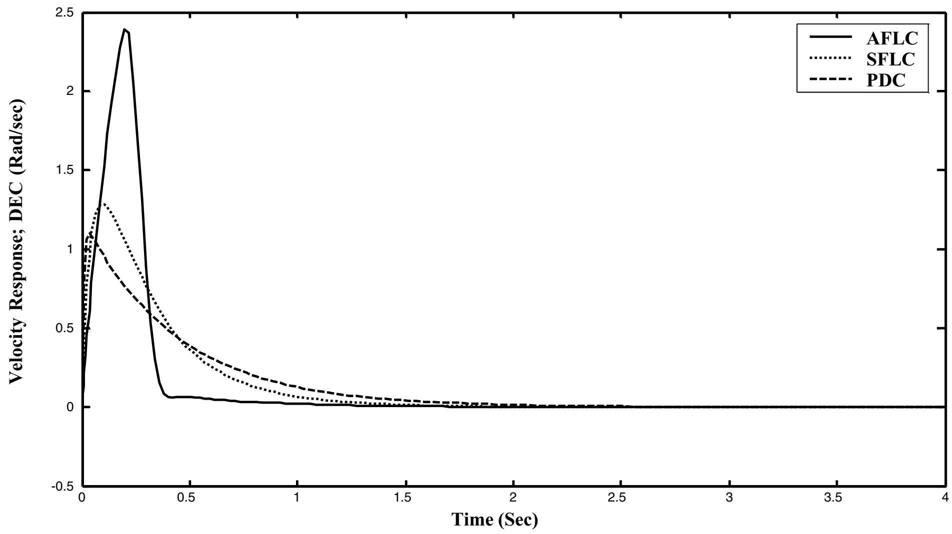

Figure 13. DEC Velocity response based on PDC, SFLC and AFLC Controllers.

desired position reference for RA and DEC, respectively. Also, the proposed AFLC controller gives a better dynamic response compared with other controller.

Therefore, the application of AFLC controller improves the dynamic response of the overall system of the astronomical telescope. It is clear that the adaptive fuzzy controlled system shows the fastest rise time and settling time with the best damping coefficient factor.

7. Conclusion

This paper presents an application of a fuzzy logic controller based on an adaptive fuzzy set to control the electric motor for position movements of the telescope. The input vectors of the adaptive tuned fuzzy logic controller are the position deviation and its rate of change. A classical fuzzy logic controller, using a static fuzzy set and rule base, has been simulated and tested. Also, the proposed FL, driven by an adaptive tuned fuzzy set, is compared with the conventional PD controller. Simulation dynamic results show the superiority of the adaptive fuzzy controller compared with other controllers used. The settling time and the rise time are decreased when using the adaptive fuzzy controller. Also, the AFLC improves the damping coefficient of the overall system under study. The simulation results show the effectiveness of the proposed FL with adaptive fuzzy set scheme as a promising technique.

Appendix. Mathematical model of the telescope

The terms of the nonlinear differential equation are defined as follows:

- **Inertia Matrix**

The inertia matrix is defined by:

$$M(\theta) = \begin{bmatrix} m_{11}(\theta) & m_{12}(\theta) \\ m_{21}(\theta) & m_{22}(\theta) \end{bmatrix}, \tag{A.1}$$

Where,

$$\begin{aligned}
m_{11}(\theta) &= I_{y1} + I_{x^2} S_2^2 + I_{y^2} C_2^2, \\
m_{12}(\theta) &= -C_1 C_2 S_2 \left(I_{x^2}(S_2 + C_1) + I_{y^2}(C_2 - S_1) \right), \\
m_{21}(\theta) &= -C_1 C_2 S_2 \left(I_{x^2}(S_2 + C_1) + I_{y^2}(C_2 - S_1) \right), \\
m_{22}(\theta) &= I_{x^2} \left(C_1^2 C_2^2 S_2^2 - 2 S_1 C_1^2 C_2^2 S_2 + S_1^2 C_1^2 C_2^2 \right), \\
&\quad + I_{y^2} \left(C_1^2 C_2^2 S_2^2 - 2 S_1 C_1^2 S_2^2 C_2 + C_1^2 S_1^2 S_2^2 \right) \\
&\quad + I_{z^2} \left(S_1^2 S_2^2 + 2 S_1 C_1^2 S_2 + C_1^4 \right).
\end{aligned}$$

- **Coriolis and Centrifugal Torque Vector**

The coriolis and centrifugal torque vector $C\left(\theta, \dot{\theta}\right)$ mentioned in equation (1) is given in the general form as follows:

$$C\left(\theta, \dot{\theta}\right) = \begin{bmatrix} b_{11}^1(\theta)\dot{\theta}_1^2 + 2b_{12}^1(\theta)\dot{\theta}_1\dot{\theta}_2 + b_{22}^1(\theta)\dot{\theta}_2^2 \\ b_{11}^2(\theta)\dot{\theta}_1^2 + 2b_{12}^2(\theta)\dot{\theta}_1\dot{\theta}_2 + b_{22}^2(\theta)\dot{\theta}_2^2 \end{bmatrix}, \tag{A.2}$$

Where,

$$
\begin{aligned}
b_{11}^1 &= 0,\\
b_{12}^1 &= C_2S_2\ \left(I_{x^2}\,(S_1S_2 +\ S_1C_1) + I_{y^2}\,(S_1C_2 -\ S_1S_1) + \left(I_{x^2}C_1S_1 + I_{y^2}C_1C_1\right)\right),\\
b_{22}^1 &= 2I_{x^2}S_2C_2 - 2I_{y^2}C_2S_2\\
&\quad + \frac{1}{2}I_{x^2}\left(2C_1C_2^2S_2^2S_1 + 2C_1^3C_2^2S_2 - 4S_1^2C_1C_2^2S_2 - 2S_1C_1^3C_2^2 + 2S_1^3C_1C_2^2\right)\\
&\quad + \frac{1}{2}I_{y^2}\ \left(2C_1C_2^2S_2^2S_1 + 2C_1^3S_2^2C_2 - 4S_1^2C_1S_2^2C_2 + 2C_1S_1^3S_2^2 - 2C_1^3S_1S_2^2\right)\\
&\quad - \frac{1}{2}I_{z^2}\left(2S_1S_2^2C_1 + 2C_1^3S_2 - 4S_1^2C_1S_2 - 4C_1^3S_1\right)\\
b_{11}^2 &= C_2S_2\left(I_{x^2}(S_1S_2 + S_1C_1) + I_{y^2}\,(S_1C_2 - S_1S_1) + \left(\left(I_{x^2}S_1C_1 + I_{y^2}C_1^2\right)\right.\right.\\
&\quad \left.\left.-\ \left(I_{y^2} - I_{x^2}\right)\right)\right)\\
b_{12}^2 &= I_{x^2}\left(-2C_1C_2^2S_2^2S_1 - 2C_1^3C_2^2S_2 + 4S_1^2C_1C_2^2S_2 + 2S_1C_1^3C_2^2 - 2S_1^3C_1C_2^2\right)\\
&\quad + I_{y^2}\left(-2C_1C_2^2S_2^2S_1 - 2C_1^3S_2^2C_2 + 4S_1^2C_1S_2^2C_2 - 2C_1S_1^3S_2^2 + 2C_1^3S_1S_2^2\right)\\
&\quad + I_{z^2}\left(2S_1S_2^2C_1 + 2C_1^3S_2 - 4S_1^2C_1S_2 - 4C_1^3S_1\right)\\
&\quad - \frac{1}{2}C_1S_2^2\left(I_2\,(S_2 + C_1) + I_{y^2}\,(C_2 - S_1)\right) + \frac{1}{2}C_1C_2^2\left(I_{x^2}\,(S_2 + C_1) + I_{y^2}\right.\\
&\quad \left.(C_2 - S_1)\right) + \frac{1}{2}C_1C_2S_2\left(I_{x^2}C_2 - I_{y^2}S_2\right)\\
b_{22}^2 &= \frac{1}{2}I_{x^2}\left(-2C_1^2C_2S_2^3 + 2C_1^2C_2^3S_2 + 4S_1C_1^2S_2^2C_2 - 2S_1C_1^2C_2^3 - 2S_1^2C_1^2C_2S_2\right)\\
&\quad + \frac{1}{2}I_{y^2}\left(-2C_1^2C_2S_2^3 + 2C_1^2C_2^3S_2 - 4S_1C_1^2C_2^2S_2 + 2S_1C_1^2S_2^3 + 2S_1^2C_1^2C_2S_2\right)\\
&\quad + \frac{1}{2}I_{z^2}\left(2S_1^2S_2C_2 + 2S_1C_1^2C_2\right)
\end{aligned}
$$

- **Gravity and loading vector**

The third term in equation (1) is defined as the gravity and loading vector $G\,(\theta)$. The general expression for $G\,(\theta)$ is given by the following equation:

$$G\,(\theta) = \begin{bmatrix} G_{11}\,(\theta) \\ G_{21}\,(\theta) \end{bmatrix}. \tag{A.3}$$

The gravity terms G_{11} and G_{21} tend to zero for a well-balanced telescope. The details of the mathematical model derivation are given in (Attia, 1997).

Notations

$[\theta_1, \theta_2]^T$	Angular positions for Right Ascension and Declination angles.
$[RA, DEC]^T$	Right Ascension and Declination angular positions.
C_1, C_2	$cos(\theta_1), cos(\theta_2)$
S_1, S_2	$sin(\theta_1), sin(\theta_2)$
I_x, I_y, I_z	The moment of inertia and subscripts of x, y, and z denote the axis about which moment of inertia is considered, respectively.
$M(\theta)$	2-dimensional matrix of inertia terms.
$C(\theta, \dot{\theta})$	Vector of centrifugal and coriolis terms.
$G(\theta)$	Vector of gravity terms.
τ	Vector of joint torques $[\tau_1, \tau_2]^T$.
Kp_1	Proportional gain of PD controller for RA position.
Kv_1	Derivative gain of PD controller for RA position.
Kp_2	Proportional gain of PD controller for DEC position.
Kv_2	Derivative gain of PD controller for DEC position.
$\theta_{1_{in}}$, and $\theta_{1_{sp}}$	Initial and final position angles of RA position.
$\theta_{2_{in}}$, and $\theta_{2_{sp}}$	Initial and final position angles of DEC position.

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