Kafrelsheikh University
Faculty of Engineering
Dept. of Mech. Engineering
Year: Third Year
Subject: Theory of vibration

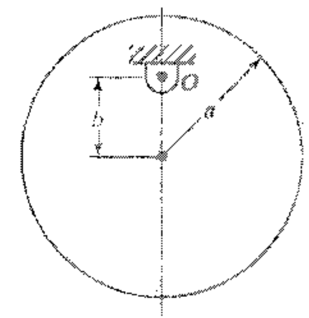
2nd Semester
Final Examination
Date: June 14, 2021
Time allowed: 3 hours
Full Mark: 100

Question #1: (20 Marks): Give brief answers to the following:

1. How do you connect several springs to increase the overall stiffness? Explain with example.
2. What effect does a decrease in mass have on the frequency of a system?
3. Is the frequency of a damped free vibration smaller or greater than the natural frequency of the system?
4. What is the use of the logarithmic decrement?
5. If a vehicle vibrates badly while moving on a uniformly bumpy road, will a change in the speed improve the condition?
6. Will the force transmitted to the base of a spring-mounted machine decrease with the addition of damping?
7. How many distinct natural frequencies can exist for an n-degree-of-freedom system?
8. How does the force transmitted to the base change as the speed of the machine increases?
9. Does the maximum amplitude equal the resonant amplitude for underdamped systems? Explain your answer.
10. If the displacement of a machine is described as $x(t) = 0.15 \sin 4t + 2.0 \cos 4t$, where x is in inches and t is in seconds, find the expressions for the velocity and acceleration of the machine. Also find the amplitudes of displacement, velocity, and acceleration of the machine.

Question # 2 (12 Marks):

A uniform circular disc is pivoted at point O, as shown in the figure. Find the natural frequency of the system. Also find the maximum frequency of the system by varying the value of b?



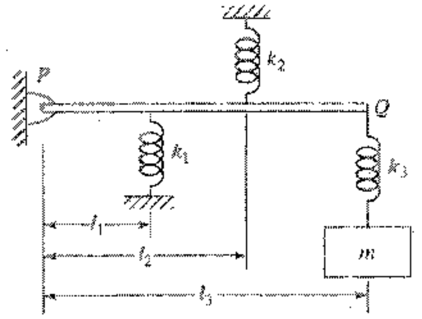
Question # 3 (12 Marks):

It was observed that the damped free oscillations of a single degree of freedom system is such that the amplitude of the tenth cycle is 40% of that of the third cycle, determine the damping factor ξ.

If the mass of the system is 4 kg and the spring constant is 800 N/m, determine the damping coefficient and the damped natural frequency.

Question # 4 (16 Marks):

Using Lagrange equation, derive the equations of motion of the free vibration of the system shown in the figure. the uniform bar PQ has a mass M and length l_3. Let the rotation angle of the bar be θ and the displacement of the mass m be x.

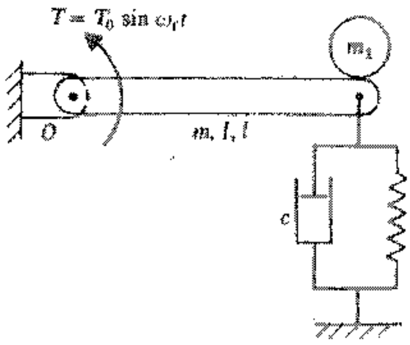


Question # 5 (20 Marks)

A damped single degree offreedom mass-spring system has a mass $m = 4$ kg, a spring stiffness $k = 2400$ N/m, and a damping coefficient c = 15 N· s/m. The mass is subjected to a harmonic force which has an amplitude $F_o = 16$ N and a frequency $\omega_f = 15$ rad/s. The initial conditions are $x_o = 4$ cm, and $\dot{x}_o = 0$. Determine the displacement, velocity, and acceleration of the mass after time $t = 0.5$ s.

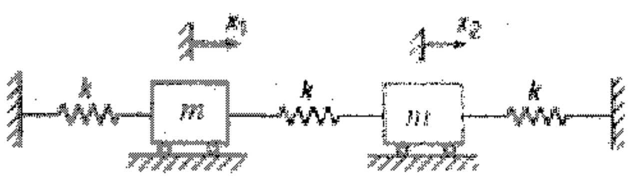
Question # 6 (20 Marks):

Assuming small angular oscillations, derive the differential equation of motion of the uniform slender beam shown in the figure. Obtain an expression for the steady state solution as a function of time.



Question # 7 (15 Marks)

Determine the natural frequencies and normal modes of the two degree of freedom system shown in the figure. Draw the normal modes.



Theory of Vibration – Formulae Sheet

Forced vibration:

$X_o = \frac{F_o}{k}$, $r = \frac{\omega_f}{\omega}$

Forced undamped vibration:

$x_p = X_o \beta \sin \omega_f t$, $\beta = \frac{1}{1-r^2}$

$x(t) = X sin(\omega t + \varphi) + X_o \beta \sin \omega_f t$

At resonance ($r = 1$):

$$x_p = -\frac{\omega X_o t}{2} \cos \omega t$$

$$x(t) = X \sin(\omega t + \varphi) - \frac{\omega X_o t}{2} \cos \omega t$$

Forced vibration of damped system:

$x_p = X_o \beta \sin(\omega_f t - \psi)$, $\psi = tan^{-1}\left(\frac{2r\xi}{1-r^2}\right)$

$\beta = \frac{1}{\sqrt{(1-r^2)^2 + (2r\xi)^2}}$, $\beta_{max.} = \frac{1}{2\xi\sqrt{1-\xi^2}}$

Force transmission:

$F_t = F_o \beta_t \sin(\omega_f t - \bar{\psi})$

Transmissibility: $\beta_t = \beta\sqrt{1 + (2r\xi)^2}$

$\bar{\psi} = \psi - \psi_t$, $\psi_t = \tan^{-1}(2r\xi)$

Work:

$W_e = \pi F_o X_o \beta \sin \psi$, $W_d = \pi c X_o^2 \beta^2 \omega_f$

$W_s = 0$

Rotating unbalance:

$$x_p(t) = \left(\frac{me}{M}\right) \beta_r \sin(\omega_f t - \psi)$$

$$\beta_r = \frac{r^2}{\sqrt{(1-r^2)^2 + (2r\xi)^2}}$$

$F_t = (me\omega^2)\beta_t \sin(\omega_f t - \bar{\psi})$

$\beta_t = \beta_r\sqrt{1 + (2r\xi)^2}$

Base motion:

$x_p(t) = Y_o \beta_b \sin(\omega_f t - \psi + \psi_b)$

$\beta_b = \frac{\sqrt{1+(2r\xi)^2}}{\sqrt{(1-r^2)^2 + (2r\xi)^2}}$, $\psi_b = \tan^{-1}(2r\xi)$

$F_t = Y_o k \beta_b r^2 \sin(\omega_f t - \psi + \psi_b)$

Relative motion:

$$z = Y_o \beta_r \sin(\omega_f t - \psi)$$

$$dB = 20Log(\frac{X_1}{X_2})$$

$$t_{p_1} = \frac{\pi / 2 - \phi}{\omega_n}$$

$$E = \frac{1}{2} k X^2$$

$$K_t = \frac{GJ}{L}$$

$\xi = \frac{c}{C_c}$, $C_C = 2m\omega$

$$X_i = \sqrt{1-\xi^2} X e^{-\xi \omega t_i}$$

logarithmic Decrement:

$$\ln \frac{X_i}{X_{i+n}} = n\xi\omega\tau_d = n\delta$$

$$\xi = \frac{\delta}{\sqrt{(2\pi)^2 + \delta^2}}$$

Energy Loss:

$$\frac{\Delta U}{U_i} = 1 - e^{-2\delta}$$

IMPACT DYNAMICS:

$m_1 v_1' + m_2 v_2' = m_1 v_1 + m_2 v_2$

$v_2' - v_1' = e(v_1 - v_2)$